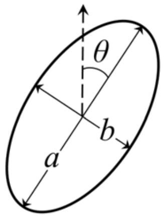
IA CHEAT SHEET

Ellipticity



$$\varepsilon = \frac{a-b}{a+b}\exp(2i\theta) \qquad \epsilon = \epsilon_1 + \mathrm{i}\epsilon_2$$

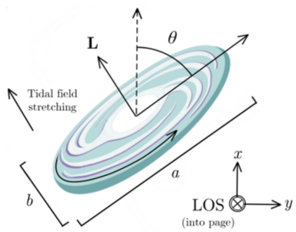
$$\chi = \frac{a^2-b^2}{a^2+b^2}\exp(2i\theta) \qquad \epsilon_1 = |\epsilon|\cos(2\theta) \qquad \epsilon_2 = |\epsilon|\sin(2\theta)$$

$$\tilde{I}_{ij} = \frac{1}{W}\sum_{k=1}^{N} w^k \frac{x_i^k x_j^k}{x^l x_l} \qquad \chi = \frac{(Q_{11}-Q_{22}, 2Q_{12})}{Q_{11}+Q_{22}+2\sqrt{\det \mathbf{Q}}}$$

$$\theta = \frac{\pi}{2} + \arctan\left(\frac{L_y}{L_x}\right)$$

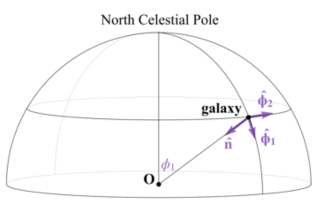
$$\frac{b}{a} = \frac{|L_{\parallel}|}{|\mathbf{L}|} + r_{\text{edge-on}}\sqrt{1-\frac{L_{\parallel}^2}{|\mathbf{L}|^2}}$$

$$T_{ij} = \frac{\partial^2\Phi}{\partial x_i \partial x_j} \qquad L_i \propto \epsilon_{ijk} I_l^k T^{jl}$$



$$\hat{\boldsymbol{\phi}}_{\mathbf{1}} = \cos\phi_1\cos\phi_2\,\hat{\mathbf{x}} + \cos\phi_1\sin\phi_2\,\hat{\mathbf{y}} - \sin\phi_1\,\hat{\mathbf{z}}\,,$$

$$\hat{\boldsymbol{\phi}}_{\mathbf{2}} = -\sin\phi_2\,\hat{\mathbf{x}} + \cos\phi_2\,\hat{\mathbf{y}}$$

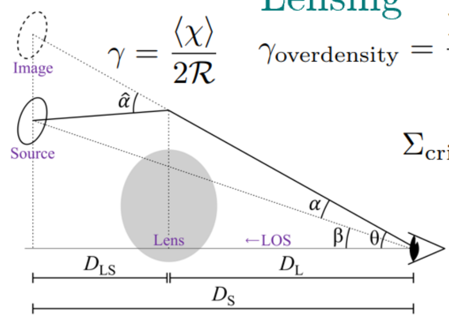


$$m_+^i = \frac{1}{\sqrt{2}}\left(\hat{\boldsymbol{\phi}}_{\mathbf{2}}^{\boldsymbol{i}} - i\hat{\boldsymbol{\phi}}_{\mathbf{1}}^{\boldsymbol{i}}\right)$$

$$m_-^i = \frac{1}{\sqrt{2}}\left(\hat{\boldsymbol{\phi}}_{\mathbf{2}}^{\boldsymbol{i}} + i\hat{\boldsymbol{\phi}}_{\mathbf{1}}^{\boldsymbol{i}}\right)$$

$$\epsilon_1 \pm i\epsilon_2 = -\frac{C_1}{4\pi G}T_{\pm} \qquad T_{\pm} = \sum_{i=1}^{3}\sum_{j=1}^{3} m^i{}_{\mp} m^j{}_{\mp} T_{ij}$$

Lensing



$$\gamma = \frac{\langle\chi\rangle}{2\mathcal{R}} \qquad \gamma_{\text{overdensity}} = \frac{\bar{\Sigma}(<r_p) - \Sigma(r_p)}{\Sigma_{\text{crit}}}$$

$$\Sigma_{\text{crit}} = \frac{c^2 D_{\rm S}}{4\pi G D_{\rm L} D_{\rm LS}}$$

$$g = \frac{\gamma}{1-\kappa}$$

$$\gamma_{\text{observed}} = \gamma_{\rm I} + \gamma_{\text{lensing}} \qquad \mu = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

$$\boldsymbol{\gamma}_3 = \boldsymbol{\gamma}_2 \oplus \boldsymbol{\gamma}_1 \iff \mathbf{S}\boldsymbol{\gamma}_3\mathbf{R} = \mathbf{S}\boldsymbol{\gamma}_2\mathbf{S}\boldsymbol{\gamma}_1$$

Correlations

$$\langle\epsilon_i\epsilon_j\rangle = \langle{\rm G}_i{\rm G}_j\rangle + \langle{\rm G}_i{\rm I}_j\rangle + \langle{\rm I}_i{\rm G}_j\rangle + \langle{\rm I}_i{\rm I}_j\rangle$$

$$\langle\epsilon_i n_j\rangle = \langle{\rm G}_i g_j\rangle + \langle{\rm I}_i g_j\rangle + \langle{\rm G}_i m_j\rangle + \langle{\rm I}_i m_j\rangle$$

$$A_+B_+ = \sum_{i\in A, j\in B}\epsilon_+(j|i)\epsilon_+(i|j) \qquad A_+B = \sum_{i\in A, j\in B}\epsilon_+(j|i)$$

$$\xi_{\times\times}(r_p,\Pi) = \frac{S_\times S_\times}{R_S R_S} \qquad \xi_{++}(r_p,\Pi) = \frac{S_+S_+}{R_S R_S}$$

$$\xi_{gg}(r_p,\Pi) = \frac{SD - R_S D - S R_D + R_S R_D}{R_S R_D}$$

$$\xi_{g+}(r_p,\Pi) = \frac{S_+D - S_+R_D}{R_S R_D} \qquad w_{ab}(r_p) = \int_{-\Pi_{\max}}^{\Pi_{\max}} \mathrm{d}\Pi\,\xi_{ab}(r_p,\Pi)$$

3D Power Spectrum

$$\gamma_{\rm I}(\boldsymbol{k}) = \gamma_1(\boldsymbol{k}) + i\gamma_2(\boldsymbol{k}) \qquad \gamma_E(\boldsymbol{k}) + i\gamma_B(\boldsymbol{k}) \equiv \gamma(\boldsymbol{k})e^{-2i\phi_{\boldsymbol{k}}}$$

$$P_{g\gamma}(\boldsymbol{k}) = \int\int \mathrm{d}^2 r_p \int \mathrm{d}\Pi\, e^{-i2\phi}e^{i\boldsymbol{k}\cdot\mathbf{r}}\xi_{g\gamma}(r_p,\Pi) = \int r_p\,\mathrm{d}r_p\int\mathrm{d}\Pi\,\xi_{g\gamma}(r_p,\Pi)\int\mathrm{d}\phi\, e^{-i2\phi}e^{ikr\cos\phi} = \int r_p\,\mathrm{d}r_p\int\mathrm{d}\Pi\,\xi_{g\gamma}(r_p,\Pi)J_2(kr_p)$$

$$(2\pi)^3\delta_D(\boldsymbol{k}-\boldsymbol{k}')P(\boldsymbol{k}) = \langle\gamma(\boldsymbol{k})\gamma^*(\boldsymbol{k}')\rangle \qquad P_{EE}(\boldsymbol{k}) = \int \mathrm{d}^2 r_p\,\mathrm{d}\Pi\, e^{-i\boldsymbol{k}\cdot\mathbf{r}}\xi_{EE}(r_p,\Pi) \qquad \xi_{g\gamma}(r_p,\Pi) = \langle g(r_p,\Pi)\gamma(r_p,\Pi)\rangle$$

$$(2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}')P_{EE}(\boldsymbol{k}) \equiv \langle\gamma_E(\boldsymbol{k})\gamma_E(\boldsymbol{k}')\rangle$$

$$(2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}')P_{\delta E}(\boldsymbol{k}) \equiv \langle\gamma_E(\boldsymbol{k})\delta_{\rm m}(\boldsymbol{k}')\rangle \qquad P_{E\delta}(\boldsymbol{k}) = \int \mathrm{d}^2 r_p\,\mathrm{d}\Pi\, e^{-i\boldsymbol{k}\cdot\mathbf{r}}\xi_{E\delta}(r_p,\Pi) \qquad \xi_{g\gamma}(\mathbf{r}) = \int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}P_{g\gamma}(\boldsymbol{k})e^{2i(\phi_{\boldsymbol{k}}-\phi_{\mathbf{r}})}e^{i\boldsymbol{k}\cdot\mathbf{r}}$$

$$P_{XY}^{(\ell)}(k) = \frac{2\ell+1}{2}\int_{-1}^{1}\mathrm{d}\mu\,\mathcal{L}_\ell(\mu)P_{XY}(k,\mu) \qquad \langle[\gamma_E(\boldsymbol{k}) + i\gamma_B(\boldsymbol{k})]g(\boldsymbol{k}')\rangle = \langle\gamma(\boldsymbol{k})g(\boldsymbol{k}')\rangle e^{-2i\phi_{\boldsymbol{k}}} \equiv (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}')P_{g\gamma}(\boldsymbol{k})$$

$$\xi_{g\gamma}(\mathbf{r}) = \int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}P_{g\gamma}(\boldsymbol{k})e^{2i(\phi_{\boldsymbol{k}}-\phi_{\mathbf{r}})}e^{i\boldsymbol{k}\cdot\mathbf{r}} \qquad \xi_{AB}(r_p,\Pi,z) = \int\frac{d^2k_\perp dk_z}{(2\pi)^3}P_{AB}(k,z)(1+\beta_A\mu^2)(1+\beta_B\mu^2)e^{i(r_pk_\perp+\Pi k_z)}$$

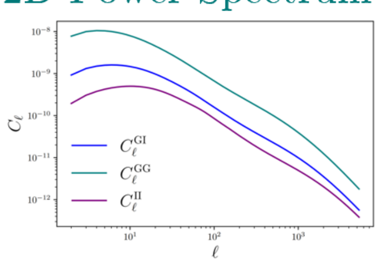
2D Power Spectrum

$$C_{\epsilon\epsilon}^{ij}(\ell) = C_{\rm GG}^{ij}(\ell) + C_{\rm GI}^{ij}(\ell) + C_{\rm II}^{ij}(\ell)$$

$$C_{\rm GG}^{ij}(\ell) = \int_0^{\chi_{\rm H}}\mathrm{d}\chi\frac{q^i(\chi)q^j(\chi)}{f_K^2(\chi)}P_{\delta\delta}\left(k=\frac{\ell}{f_K(\chi)},\chi\right)$$

$$C_{\rm GI}^{ij}(\ell) = \int_0^{\chi_{\rm H}}\mathrm{d}\chi\frac{q^i(\chi)q^j(\chi)}{f_K^2(\chi)}P_{\delta\rm I}\left(k=\frac{\ell}{f_K(\chi)},\chi\right)$$

$$C_{\rm II}^{ij}(\ell) = \int_0^{\chi_{\rm H}}\mathrm{d}\chi\frac{q^i(\chi)q^j(\chi)}{f_K^2(\chi)}P_{\rm II}\left(k=\frac{\ell}{f_K(\chi)},\chi\right)$$



$$C_\ell^{\phi\phi}(r,s) = \frac{8}{\pi}\left(\frac{3\Omega_{\rm m}^2 H_0^2}{2c^2}\right)^2\int\frac{\mathrm{d}k}{k^2}I_\ell^r(k)I_\ell^s(k)$$

$$I_\ell^r(k) = \int\frac{\mathrm{d}\chi}{\chi}[1+z(\chi)]q^r(\chi)j_\ell(k\chi)[P_\delta(k;\chi)]^{1/2}$$

$$j_\ell(k\chi) \to \sqrt{\frac{\pi}{2\nu}}\delta_D(\nu - k_\chi)$$

$$w_{g+}(r_p) = -b_g\int\mathrm{d}z\,W(z)\int_0^\infty\frac{\mathrm{d}k_\perp k_\perp}{2\pi}J_2(k_\perp r_p)P_{\delta I}(k_\perp,z)$$

$$C_\ell^{\epsilon\epsilon}(r,s) = \frac{(\ell+2)!}{\nu^4(\ell-2)!}\left(\frac{3\Omega_{\rm m}^2H_0^2}{2c^2}\right)\int\mathrm{d}\chi\,[1+z(\chi)]^2q^r(\chi)q^s(\chi)P_\delta\left(\frac{\nu}{\chi};\chi\right)$$

From *The IA Guide*, arXiv: 2309.08605

LA

Instantaneous: $A_{\rm IA}(z) = -C_1(z)\rho_{\rm m,0}(1+z)$

Early: $A_{\rm IA}(z) = -\dfrac{C_1(z)\rho_{\rm m,0}(1+z)}{\bar{D}(z)}$

$$A_{\rm IA}(z) = -C_1(z)\rho_{\rm m,0}(1+z_{\rm IA})\frac{D(z_{\rm IA})}{D(z)}$$

NLA

$$A_{\rm IA}(L,z) = A_0 \frac{C_1\rho_{\rm m,0}}{D(z)}\left(\frac{L}{L_0}\right)^{\alpha_L}\left(\frac{1+z}{1+z_0}\right)^{\alpha_z}$$

TATT

$$\gamma^{\rm I}_{ij} = C_1 s_{ij} + C_{1\delta}(\delta \times s_{ij}) + C_2\left[\sum_{k=0}^{2} s_{ik}s_{kj} - \frac{1}{3}\delta_{ij}s^2\right]$$

$$C_1(z) = -A_1(z)\frac{\bar{C}_1\rho_{\rm m}}{D(z)} \qquad C_2(z) = A_2(z)\frac{5\bar{C}_1\rho_{\rm m}}{D^2(z)}$$

Instantaneous: $A_1(z) = -C_1(z)\rho_{\rm m,0}(1+z)$

Early: $A_1(z) = -C_1(z)\rho_{\rm m,0}(1+z_{\rm IA})\dfrac{D(z_{\rm IA})}{D(z)}$

Primordial: $A_1(z) = -\dfrac{C_1(z)\rho_{\rm m,0}(1+z)}{\bar{D}(z)}$

Halo

$$P^{\rm 1h}_{\rm GI}(k) = \int {\rm d}M\, n(M)\frac{M}{\bar{\rho}_{\rm m}} f_{\rm s}\frac{\langle N_{\rm s}|M\rangle}{\bar{n}_{\rm s}}|\hat{\gamma}^{\rm I}(\boldsymbol{k}|M)|\hat{U}(M,k)$$

$$P^{\rm 2h}_{\rm GI}(k) = f_{\rm c}^{\rm red}P^{\rm red}_{\rm GI}(k) + f_{\rm c}^{\rm blue}P^{\rm blue}_{\rm GI}(k)$$

Model	Scales [h^{-1}Mpc]	Galaxy type	Study
LA	> 10	clusters	Chisari et al. (2014)
NLA	> 6	LRG	Singh et al. (2015)
TATT	> 2	LRG, ELG	Samuroff et al. (2022)
EFT	> 0.3	LRG, ELG	Bakx et al. (2023)
Halo	0.3-1.5	LRG	Singh et al. (2015)

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